Paper Reference(s)

6678

Edexcel GCE

Mechanics M2

Advanced Subsidiary

Wednesday 21 May 2008 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may gain no credit.

1. A lorry of mass 2000 kg is moving down a straight road inclined at angle α to the horizontal, where $\sin \alpha = \frac{1}{25}$. The resistance to motion is modelled as a constant force of magnitude 1600 N. The lorry is moving at a constant speed of 14 m s^{-1}.

Find, in kW, the rate at which the lorry's engine is working.

(6)

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2. A particle A of mass $4m$ is moving with speed $3u$ in a straight line on a smooth horizontal table. The particle A collides directly with a particle B of mass $3m$ moving with speed $2u$ in the same direction as A. The coefficient of restitution between A and B is e. Immediately after the collision the speed of B is $4eu$.

(a) Show that $e = \frac{3}{4}$.

(5)

(b) Find the total kinetic energy lost in the collision.

(4)

3.

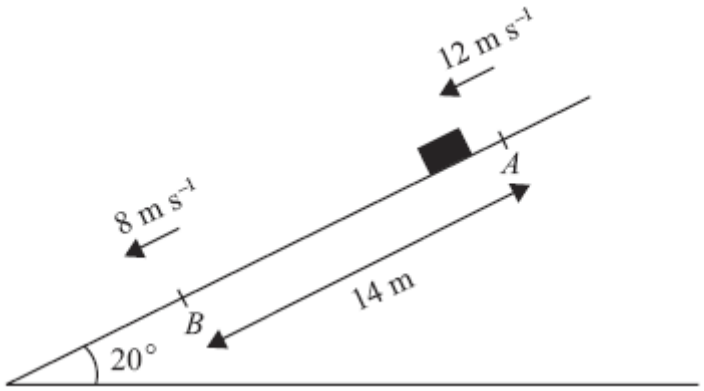

Figure 1

A package of mass 3.5 kg is sliding down a ramp. The package is modelled as a particle and the ramp as a rough plane inclined at an angle of 20° to the horizontal. The package slides down a line of greatest slope of the plane from a point A to a point B, where AB = 14 m. At A the package has speed 12 m s^{-1} and at B the package has speed 8 m s^{-1}, as shown in Figure 1.

Find

(a) the total energy lost by the package in travelling from A to B,

(5)

(b) the coefficient of friction between the package and the ramp.

(5)

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4. A particle P of mass 0.5 kg is moving under the action of a single force $\mathbf{F}$ newtons. At time t seconds,

$$\mathbf{F} = (6t - 5)\,\mathbf{i} + (t^2 - 2t)\,\mathbf{j}.$$

The velocity of P at time t seconds is $\mathbf{v}$ m s^{-1}. When $t = 0$, $\mathbf{v} = \mathbf{i} - 4\mathbf{j}$.

(a) Find $\mathbf{v}$ at time t seconds.

(6)

When $t = 3$, the particle P receives an impulse $(-5\mathbf{i} + 12\mathbf{j})$ N s.

(b) Find the speed of P immediately after it receives the impulse.

(6)

5.

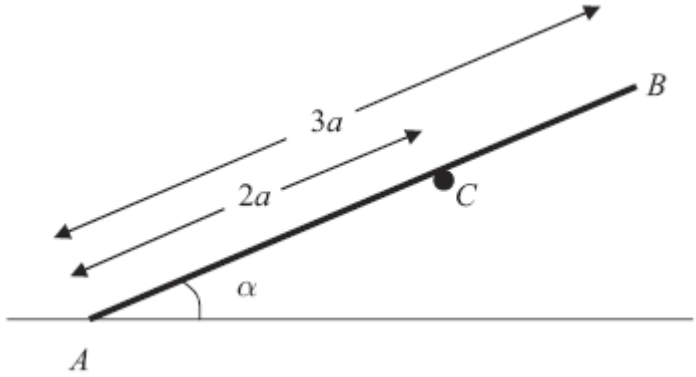


Figure 2

A plank rests in equilibrium against a fixed horizontal pole. The plank is modelled as a uniform rod AB and the pole as a smooth horizontal peg perpendicular to the vertical plane containing AB. The rod has length $3a$ and weight W and rests on the peg at C, where $AC = 2a$. The end A of the rod rests on rough horizontal ground and AB makes an angle α with the ground, as shown in Figure 2.

(a) Show that the normal reaction on the rod at A is $\frac{1}{4}(4 - 3\cos^2\alpha)W$.

(6)

Given that the rod is in limiting equilibrium and that $\cos\alpha = \frac{2}{3}$,

(b) find the coefficient of friction between the rod and the ground.

(5)

6.

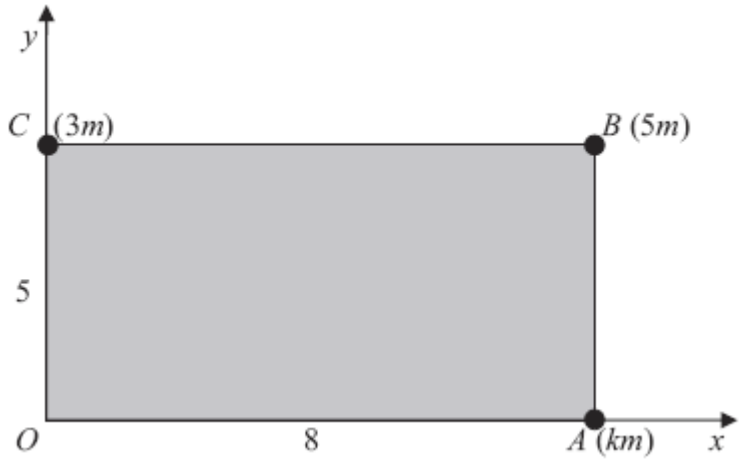


Figure 3

Figure 3 shows a rectangular lamina $OABC$. The coordinates of O, A, B and C are $(0, 0)$, $(8, 0)$, $(8, 5)$ and $(0, 5)$ respectively. Particles of mass km, $5m$ and $3m$ are attached to the lamina at A, B and C respectively.

The x-coordinate of the centre of mass of the three particles *without the lamina* is 6.4.

(a) Show that $k = 7$.

(4)

The lamina $OABC$ is uniform and has mass $12m$.

(b) Find the coordinates of the centre of mass of the combined system consisting of the three particles and the lamina.

(6)

The combined system is freely suspended from O and hangs at rest.

(c) Find the angle between OC and the horizontal.

(3)

7.

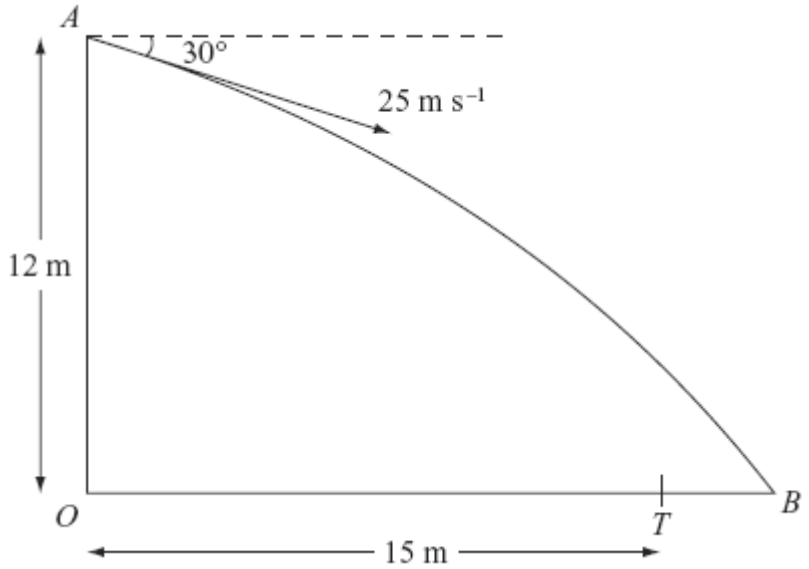


Figure 4

A ball is thrown from a point A at a target, which is on horizontal ground. The point A is 12 m above the point O on the ground. The ball is thrown from A with speed 25 m s^{-1} at an angle of 30° below the horizontal. The ball is modelled as a particle and the target as a point T. The distance OT is 15 m. The ball misses the target and hits the ground at the point B, where OTB is a straight line, as shown in Figure 4. Find

(a) the time taken by the ball to travel from A to B,

(5)

(b) the distance TB.

(4)

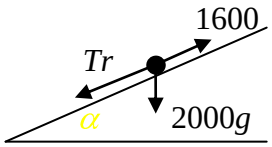
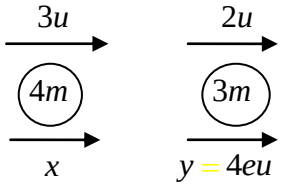
The point X is on the path of the ball vertically above T.

(c) Find the speed of the ball at X.

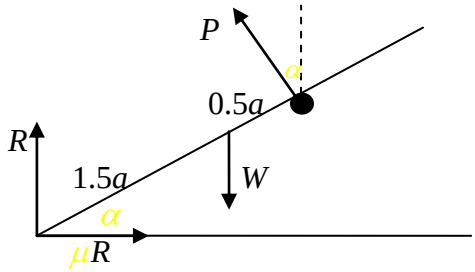
(5)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	 Resolve ↗: $T_r + 2000g \times \sin\alpha = 1600$ $T_r = 816$ $P = 816 \times 14$ W ft their T_r ≈ 11 kW accept 11.4	M1 A1 A1 M1 A1ft A1 cso (6) **(6 marks)**
2. (*a*)	 LM $12mu + 6mu = 4mx + 12meu$ NEL $4eu - x = eu$ Eliminating x to obtain equation in e Leading to $e = \frac{3}{4}$ ∗ cso	B1 M1 A1 DM1 A1 cso (5)
(*b*)	$x = 3eu$ or $\frac{9}{4}u$ or $4.5u - 3eu$ seen or implied Loss in KE $= \frac{1}{2}4m\ 3u\ ^2 + \frac{1}{2}3m\ 2u\ ^2 - \frac{1}{2}4m\left(\frac{9}{4}u\right)^2 - \frac{1}{2}3m\ 3u\ ^2$ $= 24mu^2 - 23\frac{5}{8}mu^2 = \frac{3}{8}mu^2 = 0.375mu^2$	B1 M1 A1ft A1 (4) **(9 marks)**

Question Number	Scheme	Marks
3. (*a*)	$\Delta\text{KE} = \frac{1}{2}\times 3.5\ 12^2 - 8^2 \quad = 140$ or KE at A, B correct separately	B1
	$\Delta\text{PE} = 3.5\times 9.8\times 14\sin 20° \quad \approx 164.238$ or PE at A, B correct separately	M1 A1
	$\Delta\text{E} = \Delta\text{KE} + \Delta\text{PE} \approx 304,\ \ 300$	DM1 A1 (5)
(*b*)	(b) Using Work-Energy	
	$F_r = \mu \times 3.5g\cos 20°$	M1 A1
	$304.238... = F_r \times 14$ ft their (a), F_r	M1 A1 ft
	$304.238... = \mu 3.5g\cos 20° \times 14$	
	$\mu \approx 0.674\ ,\ 0.67$	A1 (5)
		(10 marks)
4. (*a*)	N2L $6t-5\ \mathbf{i} + t^2 - 2t\ \mathbf{j} = 0.5\mathbf{a}$	M1
	$\mathbf{a} = 12t - 10\ \mathbf{i} + 2t^2 - 4t\ \mathbf{j}$	A1
	$\mathbf{v} = 6t^2 - 10t\ \mathbf{i} + \left(\frac{2}{3}t^3 - 2t^2\right)\mathbf{j} + \mathbf{C}$ ft their **a**	M1 A1ft+A1ft
	$\mathbf{v} = 6t^2 - 10t + 1\ \mathbf{i} + \left(\frac{2}{3}t^3 - 2t^2 - 4\right)\mathbf{j}$	A1 (6)
(*b*)	When $t = 3$, $\mathbf{v}_3 = 25\mathbf{i} - 4\mathbf{j}$	M1
	$-5\mathbf{i} + 12\mathbf{j} = 0.5\ \mathbf{v} - 25\mathbf{i} - 4\mathbf{j}$ ft their $\mathbf{v}_3$	M1 A1ft
	$\mathbf{v} = 15\mathbf{i} + 20\mathbf{j}$	A1
	$\lvert\mathbf{v}\rvert = \sqrt{15^2 + 20^2} = 25 \quad \text{ms}^{-1}$ cso	M1 A1 (6)
		(12 marks)

Question Number	Scheme	Marks
5. (*a*)		
	$R\uparrow \quad R+P\cos\alpha=W$	M1 A1
	$M\ A \quad P\times 2a=W\times 1.5a\cos\alpha$	M1 A1
	$\left(P=\frac{3}{4}W\cos\alpha\right)$	
	$R=W-P\cos\alpha=W-\frac{3}{4}W\cos^2\alpha$	DM1
	$=\frac{1}{4}\ 4-3\cos^2\alpha\ W\ *$ cso	A1 (6)
(*b*)	Using $\cos\alpha=\frac{2}{3}$, $\quad R=\frac{2}{3}W$	B1
	$R\rightarrow \quad \mu R=P\sin\alpha$	M1 A1
	Leading to $\mu=\frac{3}{4}\sin\alpha$	
	$\sin\alpha=\sqrt{\ 1-\frac{4}{9}\ }=\frac{\sqrt{5}}{3}$	
	$\mu=\frac{\sqrt{5}}{4}$ awrt 0.56	M1 A1 (5)
		(11 marks)

Question Number	Scheme	Marks
6. (*a*)	$M(Oy)$ $8+k\ m\times 6.4=5m\times 8+km\times 8$	M1 A1
	$1.6k=11.2 \Rightarrow k=7$ ✱ cso	M1 A1 (4)
(*b*)	$M(Oy)$ $27m\bar{x}=12m\times 4+5m\times 8+7m\times 8$	M1 A1
	$\bar{x}=\frac{16}{3}$ 5.3 or better	A1
	$M(Ox)$ $27m\bar{y}=12m\times 2.5+8m\times 5$	M1 A1
	$\bar{y}=\frac{70}{27}$ 2.6 or better	A1 (6)
(*c*)	$\tan\theta=\frac{\bar{y}}{\bar{x}}=\frac{35}{72}$	M1 A1ft
	$\theta\approx 26°$ awrt 25.9°	A1 (3)
		(13 marks)
7. (*a*)	↓ $u_y=25\sin 30°=12.5$	B1
	$12=12.5t+4.9t^2$ −1 each error	M1 A2, 1, 0
	Leading to $t=0.743$, 0.74	A1 (5)
(*b*)	→ $u_x=25\cos 30°\left(=\frac{25\sqrt{3}}{2}\approx 21.65\right)$	B1
	$OB=25\cos 30°\times t \approx 16.094\,58$ ft their (a)	M1 A1ft
	$TB\approx 1.1$ (m) awrt 1.09	A1 (4)
(*c*)	→ $15=u_x\times t\Rightarrow t=\frac{15}{u_x}\left(=\frac{2\sqrt{3}}{5}\approx 0.693 \text{ or } 0.69\right)$	M1 A1
	either ↓ $v_y=12.5+9.8t \approx 19.2896$	M1
	$V^2=u_x^{\ 2}+v_y^{\ 2} \approx 840.840$	
	$V\approx 29$ ms^{-1} , 29.0	M1 A1 (5)
		(14 marks)